Information Sciences 422 (2018) 257–270

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

On designing of an adaptive event-triggered communication scheme for nonlinear networked interconnected control systems

Zhou Gu[a,b,*], Dong Yue[b], Engang Tian[c]

[a] College of Mechanical & Electronic Engineering,Nanjing Forestry University, Nanjing 210037, China
[b] Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing 210023, China
[c] School of Electrical and Automation Engineering, Nanjing Normal University, Nanjing, Jiangsu 210023, China

ARTICLE INFO

Article history:
Received 27 October 2016
Revised 26 August 2017
Accepted 4 September 2017
Available online 6 September 2017

Keywords:
Adaptive event-triggered scheme
Nonlinear networked interconnected system
Takagi-Sugeno (T-S) fuzzy model

ABSTRACT

This paper is concerned with the design of adaptive event-triggered scheme for networked nonlinear interconnected systems via T-S fuzzy models. The transmission of control signals is based on a novel adaptive event-triggered communication scheme, where the adaptive threshold is dependent on a dynamic error of the system rather than a predetermined constant as the one in the existing results. The amount of the releasing data is regulated with the adaptive threshold that plays a crucial role in decision of whether releasing the sampled data or not. This technique reflects an inherent dynamic balance between the control performance and the utilization of the network resource. A corresponding Lyapunov function is constructed to achieve sufficient conditions of stability and stabilization. Finally, a simulation example is given to show the effectiveness of the proposed theoretic results.

1. Introduction

Many real-world systems such as economic models, spacecraft dynamics, power systems, industrial processes, and transportation networks can be modeled as interconnected systems [6,24,30,38]. Compared with the linear interconnected systems, it is more difficult to analyze the nonlinear interconnected systems due to its complex nonlinear processing. During the past several decades, there has been rapidly growing interest in the fuzzy control of nonlinear systems by using the Takagi–Sugeno (T–S) fuzzy model [4,14,16,20,26,27]. Communication network is necessary for large scale nonlinear control systems to transmit the control signals owing to the networked control systems (NCSs) with advantages of lower cost, easier maintenance and higher reliability of the closed-loop systems [2,9,18]. However, in this framework, a gap remains between the decentralized control and the network, for example, the output of the controller is a piecewise continuous-time function due to the system receiving discrete data-packets; The data can not be updated in real time on account of the network with a limited bandwidth, which decreases the control performance of the system with a network communication.

In recent years, much effort has been devoted to improve the efficiency of data releasing for NCSs, especially for wireless NCSs, such as, the NCS with Bluetooth, wireless HART and ZigBee etc., to alleviate the burden of the network-bandwidth. Periodic sampling and transmission in conventional NCSs may lead to a redundant data releasing on the grounds that the dynamical characteristics of the system is not considered in control designing. It implies that the limited resource of the

* Corresponding author.
E-mail address: gzh1808@163.com (Z. Gu).

http://dx.doi.org/10.1016/j.ins.2017.09.005
0020-0255/© 2017 Published by Elsevier Inc.

communication and computation will be occupied by some unnecessary data. Moreover, it increases the energy consumption in the implementation of data-transmission as well. For these purposes, event-based data communication scheme provides an inspiring opportunity for reducing the amount of data releasing over the network within the prescribed time period. The event of data-transmission, under this scheme, is triggered by a well-designed condition instead of a certain period of time. Less data packets are needed to transmit over the network in achieving a desired control performance compared with the time-triggered scheme.

A new task of data-transmission is generated when the state error exceeds a predetermined constant under the event-triggered scheme (ETS) . The authors in [33] designed a time-continuous ETS for distributed NCSs. The authors explored applications of self-triggered technique to distributed linear/nonlinear state feedback controllers in [1,19]. ETMs for the implementation of the observer and the output feedback controller of sampled-data systems are developed in [28]. The output feedback control of linear time invariant systems with ETS is reported in [8], where the event-triggered control system is modeled as an impulsive system. Based on passivity theory, the authors in [35] developed an event-triggering condition for the output feedback stabilizing controllers. However the aforementioned literature is based on an assumption that the controllers are known in prior. Additionally, if the error of the state varies fast, then the hardware can not reach the requirement of high-speed sampling, since the event-triggered implementation is dependent on the continuous-time state.

To solve the above mentioned problem, discrete event-triggering communication schemes for NCSs were developed in [13,21,25,31,34,36], and references therein. Under the schemes, the authors proposed some approaches to co-design both the controllers and the parameters of event-triggering. The problem of over-sampling can be avoided due to the implementation of the designed event depending on the state at discrete sampling instant. The controllers can be designed as well by using an approach of transferring the hybrid system into a delay system.

It is noted that the threshold in ETS plays a remarkable role in the decision of whether releasing the sampling data or not. Some results were given in [21,31,36] on this issue, for example, if the threshold tends to zero, the case turns to be a conventional time-triggered data releasing scheme, that is to say, the threshold taking different value may result in a distinguish difference to the amount of the data releasing. However, it is hard to predetermine a proper fixed threshold for the system in whole control process. The authors in [22,39] did some preliminary study on this issue. However, these results only depend on the upper-bound of the threshold. It is known that the adaptive control is a control method used by a controller that can adapt to the controlled system with parameters variation or uncertainty [3,7,32]. An adaptive law is needed to adapt such changing condition. Borrowing this ideas, the threshold will be regulated with the variation of external conditions by using the adaptive way. However, it is still an open and challenging issue by using an adaptive method for nonlinear interconnected systems in selecting the proper threshold. This motivates the current study.

In this paper, we proposed an adaptive event-triggered scheme (AETS) for a nonlinear networked interconnected control system. Under the proposed scheme, the releasing rate of the sampling data is dramatically reduced, by which the limited resource of the communication and computation can be allocated the other components according to the necessity of the controlled system dynamically. The main contribution of this work is as follows: Firstly, an adaptive ETS for nonlinear networked interconnected control system is proposed. The threshold of AETS is regulated on-line, which mainly depends on the dynamic error of the system. It is a result of inherent trade-off rather than a predetermined constant. Secondly, a new Lyapunov function is constructed to address the problem of nonlinearity arisen from the proposed adaptive law. Finally, sufficient conditions are derived to co-design the controllers and the parameters of AETS.

The rest of the paper is organized as follows. Section 2 presents an adaptive ETS and a unified T-S fuzzy model of nonlinear networked interconnected system under AETS. Stability analysis of the networked nonlinear interconnected systems based on AETS is provided in Section 3. Section 4 is devoted to co-design the controllers and the parameters of AETS. A simulation example is given in Section 5 to demonstrate the advantage of our adaptive event-triggering scheme. Finally, the paper is concluded in Section 6.

Notation 1. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of real $n\times m$ matrices, I is the identity matrix of appropriate dimensions, $\|\cdot\|$ stands for the Euclidean vector norm or spectral norm as appropriate. The notation $X>0$ (respectively, $X<0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite). The asterisk $*$ in a matrix is used to denote term that is induced by symmetry, Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

In this section, we will develop a framework of network-based nonlinear interconnected control systems. A novel AETS for reducing the amount of the releasing data within a given period will be proposed to mitigate the burden of the network bandwidth. Under the proposed AETS, the stability and stabilization of the networked nonlinear interconnected control system will be discussed.

2.1. The system description

Suppose a nonlinear interconnected system with time-varying coupled delay is composed of N subsystems $\mathbf{S_i}$, $i\in\{i|i=1,2,\ldots,n_s\}\triangleq\mathscr{N}$. The T-S fuzzy model of subsystem $\mathbf{S_i}$ for rule j is shown as:

Plant Rule j: IF $\theta_{i1}(t)$ is W_{i1}^j, ..., and $\theta_{ip}(t)$ is W_{ip}^j THEN

$$\mathbf{S_i}\begin{cases}\dot{x}_i(t)=A_{ij}x_i(t)+\sum_{l=1,l\neq i}^{n_s}A_{dijl}x_l(t-\eta_{il}(t))+B_{ij}u_i(t)+D_{ij}\omega_i(t) & (1)\\ z_i(t)=C_{ij}x_i(t) & (2)\\ x_i(t)=\psi_i(t)\qquad t\in[-\tilde{\eta}_{il},0] & (3)\end{cases}$$

where $j\in\{1,\dots,r_i\}\triangleq\mathscr{R}_i$ denotes the jth fuzzy inference rule; θ_{iq} and $W_{iq}^j(q=1,2,\dots,p)$ are measurable premise variables and fuzzy sets, respectively; $x_i(t)\in\mathbb{R}^{n_i}$, $u_i(t)\in\mathbb{R}^{m_i}$, $\omega_i(t)\in\mathbb{R}^{q_i}$ are the subsystem state, input vectors and the disturbance input which belongs to $\mathscr{L}_2[0,\infty)$, respectively; $\psi_i(t)$ represents the initial conditions; A_{ij}, A_{dijl}, B_{ij}, C_{ij}, and D_{ij} are matrices with appropriate dimensions; $\eta_{il}(t)$ is the coupled delay within the subsystems satisfying

$$0\le\eta_{il}(t)\le\tilde{\eta}_{il},\quad \dot{\eta}_{il}(t)\le\bar{\eta}_{il} \tag{4}$$

Denote $\theta_i(t)=[\theta_{i1}(t),\theta_{i2}(t),\dots,\theta_{ip}(t)]^T$. By using the center-average defuzzifier, product inference and singleton fuzzifier, the global dynamics of T-S fuzzy system (1) and (2) can be expressed by

$$\begin{cases}\dot{x}_i(t)=\sum_{j=1}^{r_i}h_{ij}(\theta_i(t))\left[A_{ij}x_i(t)+\sum_{l=1,l\neq i}^{n_s}A_{dijl}x_l(t-\eta_{il}(t))+B_{ij}u_i(t)+D_{ij}\omega_i(t)\right] & (5)\\ z_i(t)=\sum_{j=1}^{r_i}h_{ij}(\theta_i(t))C_{ij}x_i(t) & (6)\end{cases}$$

where

$$h_{ij}(\theta_i(t))=\frac{\mu_{ij}(\theta_i(t))}{\sum_{j=1}^{r_i}\mu_{ij}(\theta_i(t))},\quad \mu_{ij}(\theta_i(t))=\prod_{q=1}^{p}W_{iq}^j(\theta_{iq}(t))$$

with $W_{iq}^j(\theta_{iq}(t))$ being the membership value of $\theta_{iq}(t)$ in W_{iq}^j, and some basic properties of $h_{ij}(\theta_i(t))$ are

$$h_{ij}(\theta_i(t))\ge 0,\quad \sum_{j=1}^{r_i}h_{ij}(\theta_i(t))=1 \tag{7}$$

Suppose the following fuzzy decentralized controller is employed to deal with the stabilization problem of the above subsystem $\mathbf{S_i}$.

Control Rule s: IF $\theta_{i1}(t)$ is W_{i1}^s, ... and $\theta_{ip}(t)$ is W_{ip}^s THEN

$$u_i(t)=K_{is}x_i(t) \tag{8}$$

The defuzzified output of controller rule (8) is designed as

$$u_i(t)=\sum_{s=1}^{r_i}h_{is}(\theta_i(t))K_{is}x_i(t) \tag{9}$$

where K_{is} is the controller gain to be designed.

Eq. (9) denotes the control signal is transmitted by point-to-point connection, however, the control signal in NCSs is transmitted via network, based on the previous work in [11,29], the networked state feedback control law of the subsystem $\mathbf{S_i}$ can be modeled as

$$u_i(t^+)=\sum_{s=1}^{r_i}h_{is}(\theta_i(t_k^ih))K_{is}x_i(t_k^ih) \tag{10}$$

for $t\in[t_k^ih+\tau_{t_k^i},t_{k+1}^ih+\tau_{t_{k+1}^i})$, where h is a sampling period, t_k^ih denotes the releasing instant of $\mathbf{S_i}$, and $\tau_{t_k}^i$ is the transmitting delay of the packet at t_k^ih which satisfies

$$0\le\tau_{t_k}^i\le\tau_i \tag{11}$$

Here, we assume the state is available. Output feedback strategy [20] or state observer-based method [17] can be used to handle the state-unavailable case. For notational simplicity, $h_{ij}(\theta_i(t)$ and $h_{is}(\theta_i(t_k^ih))$ will be written as h_{ij} and $h_{is}^{t_k}$, respectively, in the subsequent description. Motivated by [15], we assume the premise variables satisfy $h_{is}^{t_k}=\alpha_jh_{ij}$ with $\alpha_1\le\alpha_j\le\alpha_2$.

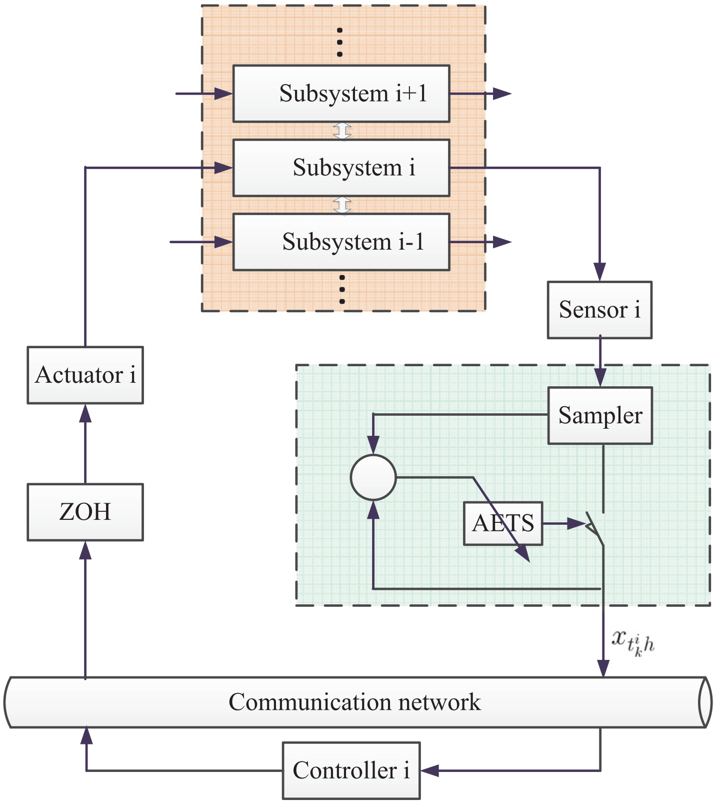

Fig. 1. The framework of networked interconnected systems with AETS.

2.2. Adaptive data releasing scheme

As is shown in Fig. 1, an adaptive event-triggered device is introduced between the sensor and the communication network in each subsystems. This device is responsible for selecting some "necessary" sampling packets for the control system to transmit over the network. The data with litter variation are regarded as "unnecessary" packets, which will be dropped out actively while the remains should be satisfied the following condition

$$(x_i(t_k^ih) - x_i(t_k^ih + \ell_{t_k}^i h))^T \Phi_i (x_i(t_k^ih) - x_i(t_k^ih + \ell_{t_k}^i h)) - \delta_i(t) x_i^T(t_k^ih)\Phi_i x_i(t_k^ih) \leq 0 \tag{12}$$

where $\Phi_i > 0$, $\ell_{t_k}^i = 1, 2, \ldots, \bar{\ell}_{t_k}^i$, and $\delta_i(t)$ is a positive time-dependent threshold function.

Define

$$\tau_{t_k^i}^s = \begin{cases} \tau_{t_k^i} & s = 0 \\ \tau_{t_{k+1}^i} & s = \bar{\ell}_{t_k}^i \\ \hat{\tau}_{t_k^i} & \text{others} \end{cases} \tag{13}$$

and $F_{t_k^i}^{\ell_{t_k}^i} = \left[t_k^ih + \ell_{t_k}^i h - h + \tau_{t_k^i}^{\ell_{t_k}^i - 1}, t_k^ih + \ell_{t_k}^i h + \tau_{t_k^i}^{\ell_{t_k}^i}\right)$, where $\hat{\tau}_{t_k}^i$ $(0 \leq \hat{\tau}_{t_k}^i \leq \tau_i)$ is an artificial constant delay to guarantee the interval $F_{t_k^i}^{\ell_{t_k}^i}$ is well defined. Obviously that $F_{t_k^i}$ is partitioned $\bar{\ell}_{t_k}^i$ intervals, and $F_{t_k^i} = \left[t_k^ih + \tau_{t_k^i}, t_{k+1}^ih + \tau_{t_{k+1}^i}\right) = \cup_{\ell_{t_k}^i = 1}^{\bar{\ell}_{t_k}^i} F_{t_k^i}^{\ell_{t_k}^i}$.

The threshold in (12) is designed as follows:

$$\dot{\delta}_i(t) = \frac{1}{\delta_i(t)}\left(\frac{1}{\delta_i(t)} - \phi_i\right)\varepsilon_{i\ell_{t_k}^i}^T(t)\Phi_i\varepsilon_{i\ell_{t_k}^i}(t) \tag{14}$$

with $0 < \delta_i(0) \leq 1$ and a given constant $\phi_i (1 \leq \phi_i)$ for $t \in F_{t_k^i}^{\ell_{t_k}^i}$, where $\varepsilon_{i\ell_{t_k}^i}(t) = x_i(t_k^ih) - x_i(t_k^ih + \ell_{t_k}^i h)$ denotes the absolute error between the state at the latest releasing instant and the one at the current sampling instant.

To better illustrate the proposed AETS, an example of time sequence of the data packet is given in Fig. 2, where '⋆', '•' and '∅' represent the releasing instant, arriving instant and the instant of the dropped-packet, respectively. In this example,

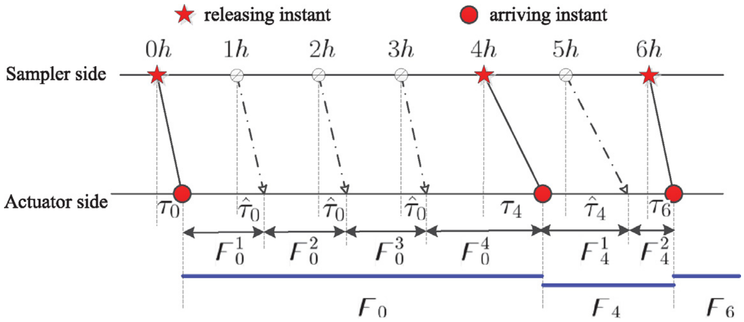

Fig. 2. An example of the sequence of AETS.

the packets at instants $0h, 4h, 6h, \ldots,$ among the periodic sampling instants are selected to transmit over the network due to theirs invoking the proposed adaptive event-triggering condition in (12), while the packets at instants $1h, 2h, 3h, 5h, \ldots,$ are dropped out actively. The maximum allowable number of successive packet losses in intervals F_0 and F_4 are $\bar{\ell}_0^i = 4$ and $\bar{\ell}_4^i = 2,$ respectively. The control input is a piecewise-continuous function owing to ZOH. Obviously, $F_0 = \cup_{\ell_0^i=1}^{4} F_0^i,$ and $\{F_0^i, F_4^i, F_6^i, \ldots\} = [t_0, \infty).$

Remark 1. It is noted that $\delta_i(t)$ has a great influence on the amount of the data-releasing in a certain time interval from the event-triggering condition in (12). $\delta_i(t)$ taking different value will result in different releasing rate of the sampling data. Specially, if one sets $\delta_i(t) \equiv 0$ (the adaptive law (14) is no longer applicable in this case), it reduces to a periodic releasing scheme since the condition (12) can not be invoked. Therefore, it is a meaningful work to develop an adaptive threshold for the event-triggering condition.

Remark 2. The proposed adaptive law in (14) is dependent on the dynamic error between the latest releasing data and the current sampling data, that is, the threshold $\delta_i(t)$ is adapted to the state variation of the system aroused from external disturbance, which is different from the existing literature on event-triggering scheme by presetting it as a constant.

Remark 3. If the system is stable, $\varepsilon^T_{i\ell^i_{t_k}}(t)\Phi_i\varepsilon_{i\ell^i_{t_k}}(t)$ in (14) finally tends to zero. As a result, the threshold $\delta_i(t)$ keeps a constant. Additionally, ϕ_i in (14) can adjust the convergence rate of $\delta_i(t)$.

Remark 4. $\hat{\tau}_{t^i_k}$ in (13) is an artificial delay. $\bar{\ell}^i_{t_k}$ is the maximum allowable number of successive packet losses of the ith system, which can be expressed by $\bar{\ell}^i_{t_k} = \sup_{\ell^i_{t_k} \geq 1} \{\ell^i_{t_k} | \varepsilon^T_{i\ell^i_{t_k}}(t)\Phi_i\varepsilon_{i\ell^i_{t_k}}(t) - \delta_i(t)x_i^T(t^i_kh)\Phi_ix_i(t^i_kh) \leq 0\}.$

From the above analysis, we can know that the next releasing instant is decided by

$$t_{k+1}h = t_kh + \bar{\ell}^i_{t_k}h \tag{15}$$

2.3. The overall model of the system based on AETS

In order to model the system (5) as a time delay system, we define

$$\rho_{i\ell^i_{t_k}}(t) = t - (t^i_kh + \ell^i_{t_k}h) \tag{16}$$

for $t \in F^{\ell^i_{t_k}}_{t^i_k}$, then it is true that

$$0 \leq \tau_i \leq \rho_{i\ell^i_{t_k}}(t) \leq \tau_i + h = \bar{\rho}_i \tag{17}$$

Combining (10) and (16), we can rewrite the control input as

$$u_i(t) = \sum_{s=1}^{r_i} h^{t_k}_{is}\Big[K_{is}\varepsilon_{i\ell^i_{t_k}}(t) + K_{is}x_i(t - \rho_{i\ell^i_{t_k}}(t))\Big] \tag{18}$$

Thus the overall closed-loop system and the triggering condition for $t \in F^{\ell^i_{t_k}}_{t^i_k}$ can be expressed by (19) and (20), respectively,

$$\dot{x}_i(t) = \sum_{j=1}^{r_i}\sum_{s=1}^{r_i} h_{ij}h^{t_k}_{is}\mathcal{A}_{ijs}\varsigma(t) \tag{19}$$

$$\varepsilon^T_{i\ell^i_{t_k}}(t)\Phi_i\varepsilon_{i\ell^i_{t_k}}(t)-\delta_i(t)x_i^T(t-\rho_{i\ell^i_{t_k}}(t))\Phi_i x_i(t-\rho_{i\ell^i_{t_k}}(t))\le 0 \tag{20}$$

where $\mathcal{A}_{ijs}=\begin{bmatrix}A_{ij} & B_{ij}K_{is} & 0 & D_{ij} & B_{ij}K_{is} & 0 & 0 & A_{di}\end{bmatrix}$, $A_{di}=[A_{dij1} \quad \dots \quad A_{diji-1} \quad A_{diji+1} \quad \dots \quad A_{dijn_s}]$, $\varsigma(t)=\text{col}\ \{\varsigma_1(t),\varsigma_2(t),\varsigma_3(t)\}$ and $\varsigma_1(t)=\text{col}\ \{x_i(t),x_i(t-\rho_{i\ell^i_{t_k}}(t)),x_i(t-\bar{\rho}_i),\omega_i(t),\varepsilon_{i\ell^i_{t_k}}(t)\}$, $\varsigma_2(t)=\text{col}\ \{\frac{1}{\rho_{i\ell^i_{t_k}}(t)}\int_{t-\rho_{i\ell^i_{t_k}}(t)}^{t}x_i^T(s)ds$, $\frac{1}{\bar{\rho}_i-\rho_{i\ell^i_{t_k}}(t)}\int_{t-\bar{\rho}_i}^{t-\rho_{i\ell^i_{t_k}}(t)}x_i^T(s)ds\}$, $\varsigma_3(t)=\text{col}\ \{x_1(t-\eta_{i1}(t)),\dots,x_{i-1}(t-\eta_{ii-1}(t)),\ x_{i+1}(t-\eta_{ii+1}(t)),\dots,x_{n_s}(t-\eta_{in_s}(t))\}$.

3. Stability analysis

In this section, we are in a position to derive a stability criterion for the networked interconnected system under the proposed AETS. Before proceeding further, the following lemma are needed.

Lemma 1. *[23] For a given matrix $R>0$, the following inequality holds for all continuously differentiable function ω in $[a,b]\to\mathbb{R}^n$:*

$\ell_R(\dot{\omega})\ge\frac{1}{b-a}(\omega(b)-\omega(a))^TR(\omega(b)-\omega(a))+\frac{3}{b-a}\tilde{\Omega}^TR\tilde{\Omega}$,

where $\ell_R(\omega)=\int_a^b\omega^T(u)R\omega(u)du$ and $\tilde{\Omega}=\omega(b)+\omega(a)-\frac{2}{b-a}\int_a^b\omega(u)du$.

Lemma 2. *[10] For given positive integers n, m, a scalar α in the interval $(0,1)$, a given $n\times n$-matrix $R>0$, two matrices W_1 and W_2 in $\mathbb{R}^{n\times m}$. Define, for all vector ξ in $\mathbb{R}^m$, the function $\Theta(\alpha,U)$ given by: $\Theta(\alpha,R)=\frac{1}{\alpha}\xi^TW_1^TRW_1\xi+\frac{1}{1-\alpha}\xi^TW_2^TRW_2\xi$. Then, if there exists a matrix X in $\mathbb{R}^{n\times n}$ such that $\begin{bmatrix}R & U\\ * & R\end{bmatrix}>0$, then the following inequality holds*

$$\min_{\alpha\in(0,1)}\Theta(\alpha,R)\ge\begin{bmatrix}W_1\xi\\W_2\xi\end{bmatrix}^T\begin{bmatrix}R & U\\ * & R\end{bmatrix}\begin{bmatrix}W_1\xi\\W_2\xi\end{bmatrix}$$

Lemma 3. *Suppose $\alpha_1\le\alpha(t)\le\alpha_2$. For any constant matrices M and N,*

$$M+\alpha(t)N<0 \tag{21}$$

holds, if and only if

$$M+\alpha_1N<0 \tag{22}$$

$$M+\alpha_2N<0 \tag{23}$$

Proof.

i) Sufficiency
For $\alpha(t)$ in (21) equals α_1 and α_2 respectively, we can obtain (22) and (23).

ii) Necessity

Case 1: $\alpha_1=\alpha_2=\alpha(t)$
It is obviously (21) is equivalent to (22) and (23) in this case.
Case 2: $\alpha_1\ne\alpha_2$
Define $f(\alpha(t))=M+\alpha(t)N$ which can be further rewritten as

$$f(\alpha(t))=\frac{\alpha_2-\alpha(t)}{\alpha_2-\alpha_1}(M+\alpha_1N)+\frac{\alpha(t)-\alpha_1}{\alpha_2-\alpha_1}(M+\alpha_1N) \tag{24}$$

It yields that $f(\alpha(t))<0$ due to (22) and (23) for all $\alpha_1\le\alpha(t)\le\alpha_2$, which means (21) holds. This completes the proof. □

For the sake of simplicity, the following symbols are introduced:

$$e_1\triangleq[I_{n\times n}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times 1}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times n(n_s-1)}],$$
$$e_2\triangleq[0_{n\times n}\ I_{n\times n}\ 0_{n\times n}\ 0_{n\times 1}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times n(n_s-1)}],$$
$$e_3\triangleq[0_{n\times n}\ 0_{n\times n}\ I_{n\times n}\ 0_{n\times 1}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times n(n_s-1)}],$$
$$e_6\triangleq[0_{n\times n}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times 1}\ 0_{n\times n}\ I_{n\times n}\ 0_{n\times n}\ 0_{n\times n(n_s-1)}],$$
$$e_7\triangleq[0_{n\times n}\ 0_{n\times n}\ 0_{n\times n}\ 0_{n\times 1}\ 0_{n\times n}\ 0_{n\times n}\ I_{n\times n}\ 0_{n\times n(n_s-1)}]$$

Theorem 1. *For given parameters γ_i, $\bar{\eta}_{il}$, $\bar{\rho}_i$, β_1, β_2, ϕ_i and matrix K_i , the networked closed-loop interconnected system (19) under AETS is asymptotically stable if there exist matrices $P_i>0$, $Q_i>0$, $Q_{il}>0$, $R_i>0$, $\Phi_i>0$ and matrix W_i with appropriate* *dimensions such that*

$$\begin{bmatrix} \Lambda_{ijj} & * & * \\ \bar{\rho}_i R_i \mathcal{A}_{ijj} & -R_i & * \\ C_{ij} e_1 & 0 & -I \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} \Lambda_{ijs} + \beta_k \Lambda_{isj} & * & * & * & * \\ \bar{\rho}_i R_i \mathcal{A}_{ijs} & -R_i & * & * & * \\ C_{ij} e_1 & 0 & -I & * & * \\ \bar{\rho}_i \beta_2 R_i \mathcal{A}_{isj} & 0 & 0 & -\beta_2 R_i & * \\ \beta_2 C_{is} e_1 & 0 & 0 & 0 & -\beta_2 I \end{bmatrix} < 0 \quad k = 1, 2 \tag{26}$$

$$\begin{bmatrix} \bar{R}_i & * \\ W_i & \bar{R}_i \end{bmatrix} > 0 \tag{27}$$

hold for $i, j \in \mathscr{R}_i$, $i < j$, *where*

$$\begin{aligned} \Lambda_{ijs} &= diag\left\{\mathbb{Q}_i, \Phi_i, -Q_i, -\gamma_i^2 I, -\phi_i \Phi_i, 0, 0, \bar{Q}_{il}\right\} \\ &\quad + e_1^T P_i \mathcal{A}_{ijs} + \mathcal{A}_{ijs}^T P_i e_1 - \begin{bmatrix} \Xi_1 \\ \Xi_2 \end{bmatrix}^T \begin{bmatrix} \bar{R}_i & * \\ W_i & \bar{R}_i \end{bmatrix} \begin{bmatrix} \Xi_1 \\ \Xi_2 \end{bmatrix}, \\ \Xi_1 &= \begin{bmatrix} e_1 - e_2 \\ e_1 + e_2 - 2e_6 \end{bmatrix}, \Xi_2 = \begin{bmatrix} e_2 - e_3 \\ e_2 + e_3 - 2e_7 \end{bmatrix}, \\ \bar{R}_i &= diag\{R_i, 3R_i\}, \mathbb{Q}_i = \left(Q_i + \sum_{l=1, l\neq i}^{n_s} Q_{li}\right), \\ \bar{Q}_{il} &= diag\{(1-\bar{\eta}_{i1})Q_{i1}, \ldots, (1-\bar{\eta}_{ii-1})Q_{ii-1}, \\ &\qquad (1-\bar{\eta}_{ii+1})Q_{ii+1}, \ldots, (1-\bar{\eta}_{in_s})Q_{in_s}\} \end{aligned}$$

Proof. Construct a Lyapunov–Krasovskii functional candidate for the system (19) as

$$V(t) = \sum_{i=1}^{n_s} (V_{1i}(t) + V_{2i}(t) + V_{3i}(t) + V_{4i}(t)) \tag{28}$$

where

$$\begin{aligned} V_{1i}(t) &= x_i^T(t) P_i x_i(t) \\ V_{2i}(t) &= \int_{t-\bar{\rho}_i}^{t} x_i^T(s) Q_i x_i(s) ds + \sum_{l=1, l\neq i}^{n_s} \int_{t-\eta_{il}(t)}^{t} x_l^T(s) Q_{il} x_l(s) ds \\ V_{3i}(t) &= \bar{\rho}_i \int_{\bar{\rho}_i}^{0} \int_{t-s}^{t} \dot{x}_i^T(v) R_i \dot{x}_i(v) dv ds \\ V_{4i}(t) &= \frac{1}{2} \delta_i^2(t) \end{aligned}$$

Taking the time derivative along trajectories of (19), we have

$$\dot{V}_{1i}(t) = \sum_{j=1}^{r_i} \sum_{s=1}^{r_i} h_{ij} h_{is}^{t_k} 2x_i^T(t) P_i \mathcal{A}_{ijs} \varsigma(t) \tag{29}$$

$$\begin{aligned} \dot{V}_{2i}(t) &= x_i^T(t) Q_i x_i(t) + \sum_{l=1, l\neq i}^{n_s} x_l^T(t) Q_{il} x_l(t) - x_i^T(t-\bar{\rho}_i) Q_i x_i(t-\bar{\rho}_i) \\ &\quad - \sum_{l=1, l\neq i}^{n_s} (1-\dot{\eta}_{il}) x_l^T(t-\eta_{il}(t)) Q_{il} x_l(t-\eta_{il}(t)) \end{aligned} \tag{30}$$

From Lemma (1), we have

$$-\bar{\rho}_i \int_{t-\bar{\rho}_i}^{t} \dot{x}_i^T(s) R_i \dot{x}_i(s) ds = -\bar{\rho}_i \int_{t-\rho_{i\iota_{t_k}^i}(t)}^{t} \dot{x}_i^T(s) R_i \dot{x}_i(s) ds - \bar{\rho}_i \int_{t-\bar{\rho}_i}^{t-\rho_{i\iota_{t_k}^i}(t)} \dot{x}_i^T(s) R_i \dot{x}_i(s) ds$$

$$\leq -\varsigma^T(t)\left\{\frac{1}{\beta_i(t)}(e_1-e_2)^T R_i(e_1-e_2)+\frac{3}{\beta_i(t)}(e_1+e_2-2e_6)^T R_i(e_1+e_3-2e_6)\right\}\varsigma(t)$$
$$-\varsigma^T(t)\left\{\frac{1}{1-\beta_i(t)}(e_2-e_3)^T R_i(e_2-e_3)+\frac{3}{1-\beta_i(t)}(e_2+e_3-2e_7)^T R_i(e_2+e_3-2e_7)\right\}\varsigma(t)$$
$$=-\frac{1}{\beta_i(t)}\varsigma^T(t)\Xi_1^T\bar{R}_i\Xi_1\varsigma^T(t)+\frac{1}{1-\beta_i(t)}\varsigma^T(t)\Xi_2^T\bar{R}_i\Xi_2\varsigma^T(t)$$

where $0\leq\beta_i(t)=\rho_{i\ell^i_{t_k}}(t)/\bar{\rho}_i\leq 1$.

Applying Lemma 2, we can obtain

$$-\bar{\rho}_i\int_{t-\bar{\rho}_i}^{t}\dot{x}_i^T(s)R_i\dot{x}_i(s)ds\leq-\varsigma^T(t)\begin{bmatrix}\Xi_1\\ \Xi_2\end{bmatrix}^T\begin{bmatrix}\bar{R}_i & *\\ W_i & \bar{R}_i\end{bmatrix}\begin{bmatrix}\Xi_1\\ \Xi_2\end{bmatrix}\varsigma(t) \tag{31}$$

Taking the time derivative for $V_{4i}(t)$ yields

$$\begin{aligned}\dot{V}_{4i}(t)&=\delta_i(t)\dot{\delta}_i(t)\\ &=\delta_i(t)\frac{1}{\delta_i(t)}\left(\frac{1}{\delta_i(t)}-\phi_i\right)\varepsilon_i^T(t)\Phi_i\varepsilon_i(t)\\ &=\frac{1}{\delta_i(t)}\varepsilon_i^T(t)\Phi_i\varepsilon_i(t)-\phi_i\varepsilon_i^T(t)\Phi_i\varepsilon_i(t)\\ &\leq x_i^T(t-\rho_{i\ell^i_{t_k}}(t))\Phi_i x_i(t-\rho_{i\ell^i_{t_k}}(t))-\phi_i\varepsilon_i^T(t)\Phi_i\varepsilon_i(t)\end{aligned} \tag{32}$$

For the class of interconnected systems (19), the following structural identity holds:

$$\sum_{i=1}^{n_s}\sum_{l=1}^{n_s}x_l^T(t)Q_{il}x_l(t)=\sum_{l=1}^{n_s}\sum_{i=1}^{n_s}x_i^T(t)Q_{li}x_i(t) \tag{33}$$

Combining (29)–(33), we can obtain

$$\begin{aligned}\dot{V}(t)\leq&\sum_{i=1}^{n_s}\sum_{j=1}^{r_i}\sum_{s=1}^{r_i}h_{ij}h_{is}^{t_k}\Big\{2x_i^T(t)P_i\mathcal{A}_{ijs}\varsigma(t)+x_i^T(t)Q_i x_i(t)\\ &+x_i^T(t)\sum_{l=1,l\neq i}^{n_s}Q_{li}x_i(t)-x_i^T(t-\bar{\rho}_i)Q_i x_i(t-\bar{\rho}_i)\\ &-\sum_{l=1}^{n_s}(1-\bar{\eta}_{il})x_l^T(t-\eta_{il}(t))Q_{il}x_l(t-\eta_{il}(t))\\ &+\varsigma^T(t)\bar{\rho}_i^2\mathcal{A}_{ijs}^T R_i\mathcal{A}_{ijs}\varsigma(t)-\varsigma^T(t)\begin{bmatrix}\Xi_1\\ \Xi_2\end{bmatrix}^T\begin{bmatrix}\bar{R}_i & *\\ W_i & \bar{R}_i\end{bmatrix}\begin{bmatrix}\Xi_1\\ \Xi_2\end{bmatrix}\varsigma(t)\\ &+x_i^T(t-\rho_{i\ell^i_{t_k}}(t))\Phi_i x_i(t-\rho_{i\ell^i_{t_k}}(t))-\phi_i\varepsilon_i^T(t)\Phi_i\varepsilon_i(t)\\ &+x_i^T(t)C_{ij}^T C_{ij}x_i(t)-\gamma_i^2\omega_i^T(t)\omega_i(t)\\ &-z_i^T(t)z_i(t)+\gamma^2\omega_i^T(t)\omega_i(t)\Big\}\end{aligned}$$

Then we have

$$\dot{V}(t)+z_i^T(t)z_i(t)-\gamma_i^2\omega_i^T(t)\omega_i(t)\leq\sum_{i=1}^{n_s}\sum_{j=1}^{r_i}\sum_{s=1}^{r_i}h_{ij}h_{is}^{t_k}\varsigma_i^T(t)(\Lambda_{ijs}+\bar{\Lambda}_{ijs})\varsigma_i(t) \tag{34}$$

where $\bar{\Lambda}_{ijs}=\bar{\rho}_i^2\mathcal{A}_{ijs}^T R_i\mathcal{A}_{ijs}+e_1^T C_{ij}^T C_{ij}e_1$. Due to the assumption that $h_{is}^{t_k}=\alpha_s h_{ij}$, it follows that

$$\begin{aligned}\dot{V}(t)+z_i^T(t)z_i(t)-\gamma_i^2\omega_i^T(t)\omega_i(t)\leq&\sum_{i=1}^{n_s}\sum_{j=1}^{r_i}\alpha_s h_{ij}^2\varsigma_i^T(t)(\Lambda_{ijj}+\bar{\Lambda}_{ijj})\varsigma(t)\\ &+\sum_{i=1}^{n_s}\sum_{j=1}^{r_i-1}\sum_{j>s}^{r_i}h_{ij}h_{is}\varsigma_i^T(t)\big[\alpha_s(\Lambda_{ijs}+\bar{\Lambda}_{ijs})+\alpha_j(\Lambda_{isj}+\bar{\Lambda}_{isj})\big]\varsigma(t)\end{aligned}$$

By using Shur complement, one can know that (35) and (36) are equivalent to (25) and (26), respectively.

$$\bar{\Lambda}_{ijs}+\beta_2\bar{\Lambda}_{isj}+\Lambda_{ijs}+\beta_1\Lambda_{isj}<0 \tag{35}$$

$$\bar{\Lambda}_{ijs} + \beta_2 \bar{\Lambda}_{isj} + \Lambda_{ijs} + \beta_2 \Lambda_{isj} < 0 \tag{36}$$

Define $\alpha_{js} = \frac{\alpha_j}{\alpha_s}$, $\beta_1 = \frac{\alpha_1}{\alpha_2}$ and $\beta_2 = \frac{\alpha_2}{\alpha_1}$. We have $\beta_1 \le \alpha_{js} \le \beta_2$. It yields that

$$\bar{\Lambda}_{ijs} + \beta_2 \bar{\Lambda}_{isj} + \Lambda_{ijs} + \alpha_{js} \Lambda_{isj} < 0$$

from Lemma 3, which is a sufficient condition to guarantee

$$\bar{\Lambda}_{ijs} + \alpha_{js} \bar{\Lambda}_{isj} + \Lambda_{ijs} + \alpha_{js} \Lambda_{isj} < 0 \tag{37}$$

due to $\bar{\Lambda}_{isj} \ge 0$. Then we can conclude that (25) and (26) are sufficient conditions to guarantee

$$\dot{V}(t) \le -z_i^T(t) z_i(t) + \gamma_i^2 \omega_i^T(t) \omega_i(t) \tag{38}$$

Integrating of both sides of the above equation from 0 to t yields

$$V(t) - V(0) + \int_0^t z_i^T(t) z_i(t) - \int_0^t \gamma_i^2 \omega_i^T(t) \omega_i(t) \le 0 \tag{39}$$

Under zero initial condition, let $t \to \infty$, it follows

$$\int_0^\infty z_i^T(t) z_i(t) \le \int_0^\infty \gamma_i^2 \omega_i^T(t) \omega_i(t) \tag{40}$$

that is $\|z_i(t)\|_2 \le \gamma_i \|\omega_i(t)_2\|$.

With the condition of $\omega_i(t) = 0$, we can conclude that $\dot{V}(t) < 0$ from Eq. (38). Thus the proof is completed. □

Remark 5. By constructing a novel item $V_4(t)$ in Lyapunov function (28), the adaptive law in (14) is then well fit in deriving the criteria to resolve the nonlinear problem.

4. Controller design based on AETS

Based on Theorem 1, we are now ready to solve the control design problem based on AETS for the networked interconnected system.

Theorem 2. *For given parameters $\gamma_i, \bar{\eta}_{il}, \bar{\rho}_i, \beta_1, \beta_2$ and ϕ_i, the networked closed-loop interconnected system (19) under AETS is asymptotically stable if there exist matrices $X_i > 0, \tilde{Q}_i > 0, \tilde{Q}_{il} > 0, \tilde{R}_i > 0$ and $\tilde{\Phi}_i > 0$ $(i \in \mathcal{N}, i \ne l)$ and matrices W_i, Y_{is} $(i \in \mathcal{N}, i \ne l)$ with appropriate dimensions such that*

$$\begin{bmatrix} \tilde{\Lambda}_{ijj} & * & * \\ \bar{\rho}_i \mathscr{A}_{ijj} & -2\sigma_i X_i + \sigma_i^2 \tilde{R}_i & * \\ C_{ij} X_i e_1 & 0 & -I \end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix} \tilde{\Lambda}_{ijs} + \beta_k \tilde{\Lambda}_{isj} & * & * & * & * \\ \bar{\rho}_i \mathscr{A}_{ijs} & -2\sigma_i X_i + \sigma_i^2 \tilde{R}_i & * & * & * \\ C_{ij} X_i e_1 & 0 & -I & * & * \\ \bar{\rho}_i \beta_2 \mathscr{A}_{isj} & 0 & 0 & -2\sigma_i \beta_2 X_i + \sigma_i^2 \beta_2 \tilde{R}_i & * \\ \beta_2 C_{is} X_i e_1 & 0 & 0 & 0 & -\beta_2 I \end{bmatrix} < 0 \quad k = 1, 2 \tag{42}$$

$$\begin{bmatrix} \bar{R}_i & * \\ W_i & \bar{R}_i \end{bmatrix} > 0 \tag{43}$$

hold $i, j \in \mathscr{R}_i, i < j$, where

$$\tilde{\Lambda}_{ijs} = diag\left\{\tilde{\mathbb{Q}}_i, \tilde{\Phi}_i, -\tilde{Q}_i, -\gamma_i^2 I, -\phi_i \tilde{\Phi}_i, 0, 0, \tilde{Q}_{il}\right\} + e_1^T \mathscr{A}_{ijs} + \mathscr{A}_{ijs}^T e_1 - \begin{bmatrix} \Xi_1 \\ \Xi_2 \end{bmatrix}^T \begin{bmatrix} \tilde{\tilde{R}}_i & * \\ \tilde{W}_i & \tilde{\tilde{R}}_i \end{bmatrix} \begin{bmatrix} \Xi_1 \\ \Xi_2 \end{bmatrix},$$

$$\tilde{\mathbb{Q}}_i = \tilde{Q}_i + \sum_{l=1, l \ne i}^{n_s} \tilde{Q}_{li},$$

$$\bar{Q}_{il} = diag\{(1-\bar{\eta}_{i1})\tilde{Q}_{i1}, \ldots, (1-\bar{\eta}_{ii-1})\tilde{Q}_{ii-1}, (1-\bar{\eta}_{ii+1})\tilde{Q}_{ii+1}, \ldots, (1-\bar{\eta}_{in_s})\tilde{Q}_{in_s}\},$$

$$\mathscr{A}_{ijs} = \begin{bmatrix} A_{ij}X_i & B_{ij}Y_{is} & 0 & D_{ij} & B_{ij}Y_{is} & 0 & 0 & \tilde{A}_{di} \end{bmatrix}$$
$$\tilde{A}_{di} = \begin{bmatrix} A_{dij1}X_1 & \dots & A_{diji-1}X_{i-1} & A_{diji+1}X_{i+1} & \dots A_{dijn_s}X_{n_s} \end{bmatrix},$$
$$\tilde{\tilde{R}}_i = diag\{\tilde{R}_i, 3\tilde{R}_i\}$$

Proof. Eq. (25) is equivalent to

$$\begin{bmatrix} \Lambda_{ijj} & * & * \\ \bar{\rho}_i \mathcal{A}_{ijj} & -P_i R_i^{-1} P & * \\ C_{ij} e_1 & 0 & -I \end{bmatrix} < 0 \tag{44}$$

It is true that

$$-P_i R_i^{-1} P_i \le -2\sigma_i P_i + \sigma_i^2 R_i \tag{45}$$

due to $(\sigma_i R_i - P_i) R_i^{-1} (\sigma_i R_i - P_i) \ge 0$, where σ_i is a positive scalar.

From (45), we can conclude that Eq. (46) is a sufficient condition to guarantee (44) holds.

$$\begin{bmatrix} \Lambda_{ijj} & * & * \\ \bar{\rho}_i P_i \mathcal{A}_{ijj} & -2\sigma_i P_i + \sigma^2 R_i & * \\ C_{ij} e_1 & 0 & -I \end{bmatrix} < 0 \tag{46}$$

For $i \in \mathcal{N}$, we define $X_i = P_i^{-1}$, $\tilde{Q}_i = X_i Q_i X_i$, $\tilde{Q}_{li} = X_i Q_{li} X_i$, $\tilde{R}_i = X_i R_i X_i$, $\tilde{\Phi}_i = X_i \Phi_i X_i$, $Y_{is} = K_{is} X_i$, $J_1 = \text{diag}\{X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_{n_s}\}$, $J_2 = \text{diag}\{X_i, I\}$, $J_3 = \text{diag}\{X_i, X_i, X_i, I, X_i, X_i, X_i, J_1, J_2\}$, and $J_4 = \text{diag}\{X_i, X_i\}$. Pre- and post-multiplying (46) and (27) with J_3 and J_4, respectively, one can obviously know that (41) and (43) are equivalent to (46) and (27), respectively. Similarly, we can obtain the result that Eq. (42) is a sufficient condition to guarantee Eq. (26) holds. This completes the proof. □

5. A simulation example

In this section, an example will be presented to show the effectiveness of the proposed scheme. Consider a balancing double-inverted pendulums connected by a torsional spring [37] which can be modeled by T-S fuzzy model. The parameters of the model is as follows:

$$A_{11} = \begin{bmatrix} 0 & 1 \\ -0.6576 & 0 \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & 1 \\ 8.8100 & 0 \end{bmatrix}, A_{13} = \begin{bmatrix} 0 & 1 \\ -0.6576 & 0 \end{bmatrix},$$
$$A_{d112} = A_{d122} = A_{d132} = \begin{bmatrix} 0 & 0 \\ 0.8 & 0 \end{bmatrix}, B_{11} = B_{12} = B_{13} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$
$$C_{11} = C_{12} = C_{13} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{11} = D_{12} = D_{13} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0 & 1 \\ -0.4576 & 0 \end{bmatrix}, A_{22} = \begin{bmatrix} 0 & 1 \\ 9.01 & 0 \end{bmatrix}, A_{23} = \begin{bmatrix} 0 & 1 \\ -0.4576 & 0 \end{bmatrix},$$
$$A_{d211} = A_{d221} = A_{d231} = \begin{bmatrix} 0 & 0 \\ 0.8 & 0 \end{bmatrix}, B_{21} = B_{22} = B_{23} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix},$$
$$C_{21} = C_{22} = C_{23} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{21} = D_{22} = D_{23} = \begin{bmatrix} 0 \\ 0.4 \end{bmatrix}$$

The normalized membership functions of the two subsystems are chosen to be the same with the following format which is shown in Fig. 3.

$$h_{i1} = \begin{cases} 1 & x_{i1} \le -86 \\ -\dfrac{1}{81}(x_{i1} + 5) & -86 < x_{i1} \le -5, \\ 0 & \text{others} \end{cases}$$
$$h_{i2} = \begin{cases} 1 & |x_{i1}| \le 5 \\ \dfrac{1}{81}(86 - |x_{i1}|) & 5 < |x_{i1}| \le 86 \\ 0 & \text{others} \end{cases}$$

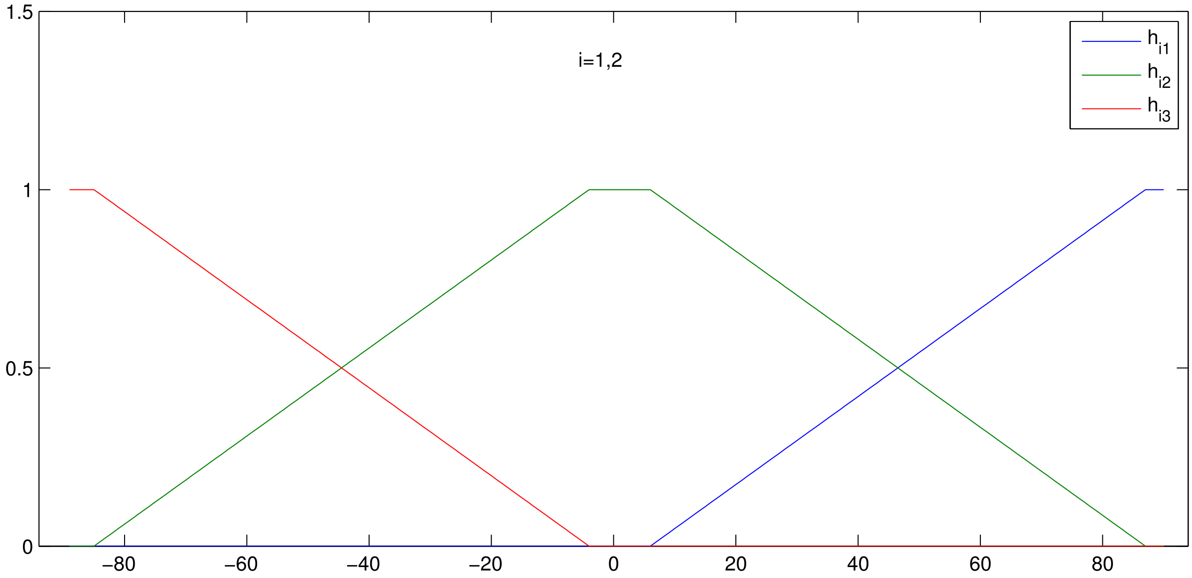

Fig. 3. Normalized membership function.

$$h_{i3} = \begin{cases} 0 & x_{i1} \le 5 \\ \frac{1}{81}(x_{i1} - 5) & 5 < x_{i1} \le 86 \\ 1 & \text{others} \end{cases}$$

For given $\phi_1 = \phi_2 = 5$ and $\bar{\rho}_1 = \bar{\rho}_2 = 0.03, \bar{\eta}_{ij} = 0.2;\ \gamma_1 = \gamma_2 = 10,$ by using Theorem 2 we can obtain the following controller gains

$$K_{11} = \begin{bmatrix} -34.8721 & -15.2665 \end{bmatrix}, K_{12} = \begin{bmatrix} -49.3824 & -15.8836 \end{bmatrix}, K_{13} = \begin{bmatrix} -35.7217 & -15.6003 \end{bmatrix},$$
$$K_{21} = \begin{bmatrix} -43.9872 & -19.0871 \end{bmatrix}, K_{22} = \begin{bmatrix} -61.9485 & -19.8560 \end{bmatrix}, K_{23} = \begin{bmatrix} -45.1869 & -19.4309 \end{bmatrix}$$

and the parameters of AETS as follows:

$$\Phi_1 = \begin{bmatrix} 0.8771 & -2.4900 \\ -2.4900 & 19.4041 \end{bmatrix}, \Phi_2 = \begin{bmatrix} 0.9206 & -2.6146 \\ -2.6146 & 19.4529 \end{bmatrix}. \tag{47}$$

To illustrate the validity of the proposed AETS, in the following, we will present the simulation results with the initial states $\psi_1(t) = \begin{bmatrix} -0.9 & 0.6 \end{bmatrix}, \psi_2(t) = \begin{bmatrix} 0.1 & -0.4 \end{bmatrix}$, and sampling period $h = 0.02$.

From Fig. 5, one can see that (1) large plenty of sampling data are dropped out due to the implementation of AETS; and (2) the releasing rate of sampling data before 3s is obviously bigger than the one after 3s since the state of system has a bigger variation before it tends to stable. Although the system has some unstable poles, it can be well stabilized as well under the proposed AETS, which is shown in Fig. 4.

The responses of adaptive threshold are depicted in Fig. 7, from which one can see that the thresholds $\delta_i(t)\ (i = 1, 2)$ finally converge to 0.416 and 0.118, respectively. The threshold is adaptively regulated with the state variation rather than a predetermined constant. Under this threshold, the rate of data-releasing can meet the necessity of control performance dynamically.

If one sets the threshold as a constant, i.e. $\delta_1(t) = 0.416$ and $\delta_1(t) = 0.118$, it will be reduced to a conventional ETS. The triggering instants and their corresponding releasing intervals of the system with a fixed threshold are plotted in Fig. 6. Theoretically, the releasing rate of sampling data during the unstable period should be bigger than the one during the stable period. From this point of view, one can conclude that the proposed AETS is much better than the conventional event-triggered method by comparing Fig. 5 with Fig. 6.

6. Conclusion

In this paper, an adaptive ETS for nonlinear networked interconnected control system based on T-S fuzzy model is developed. The threshold of the event-triggering condition is adapted with dynamic error of the system including the external disturbance, which is different from the conventional way by using a predetermined constant. A co-design method is given to achieve the fuzzy controllers and the parameters of the triggering condition. A simulation example has been shown that

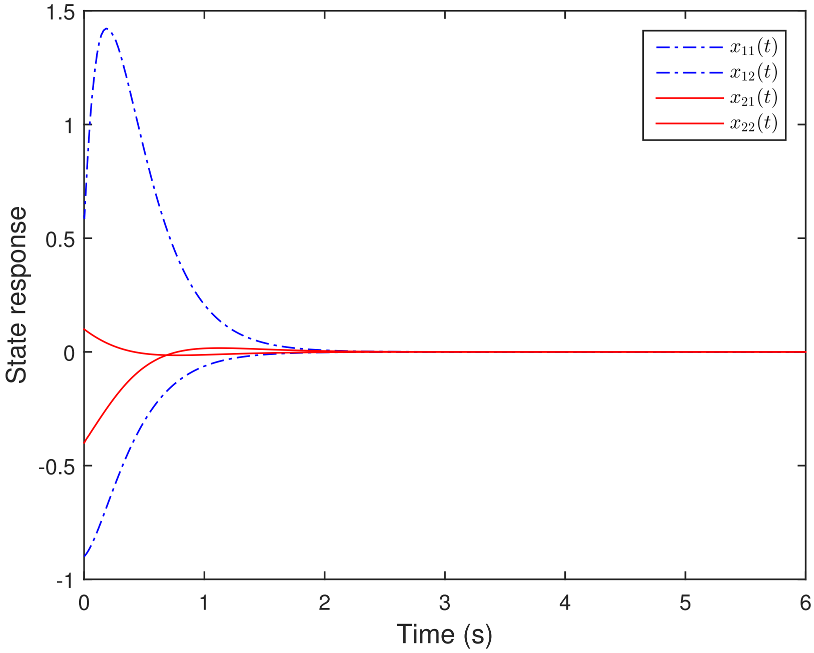

Fig. 4. The states of the system based on AETS.

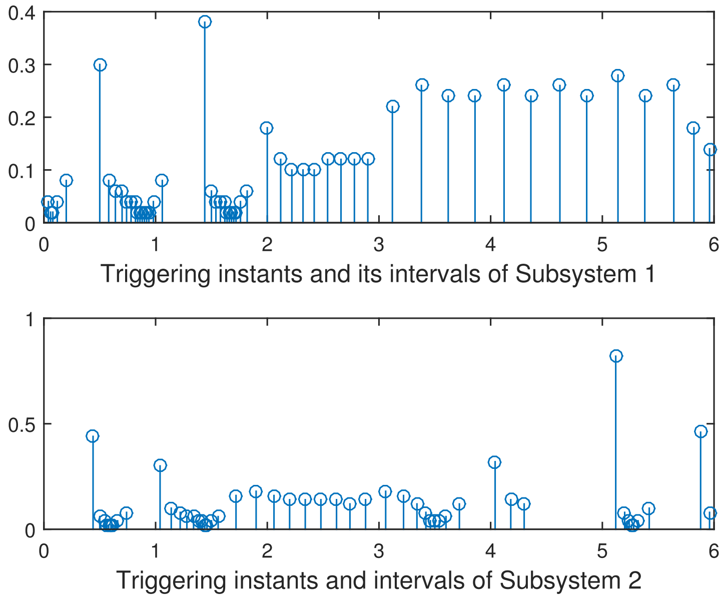

Fig. 5. The triggering instants and the releasing intervals with AETS.

the threshold is an optimal result and the network resource can be saved by the proposed method. Introducing some novel approaches, such as the method of integral inequalities used in [5,12] may result in less conservative results, which will be investigated in the future.

Acknowledgement

This work was supported by the National Science Foundation of China (Grant 61473156, 61773218, 61640313, 61403185), Research Fund for the Doctoral Program of Higher Education of China (Grant. 20133204120018).

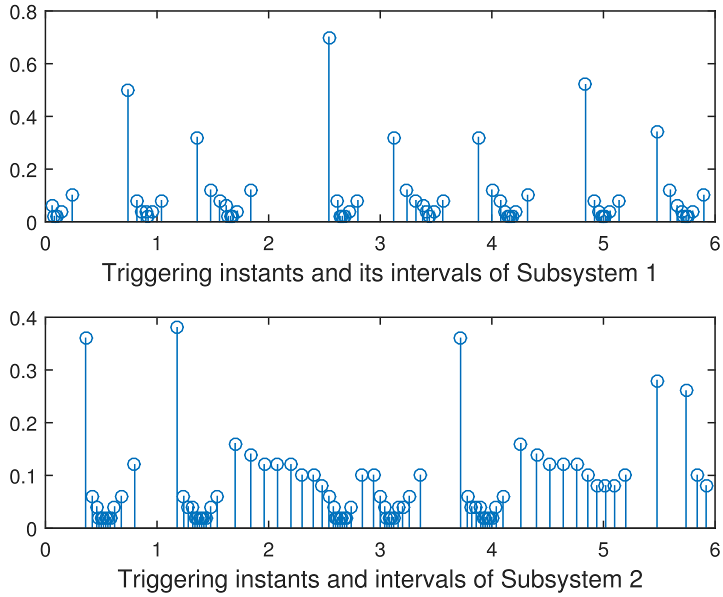

Fig. 6. The triggering instants and the releasing intervals with fixed thresholds.

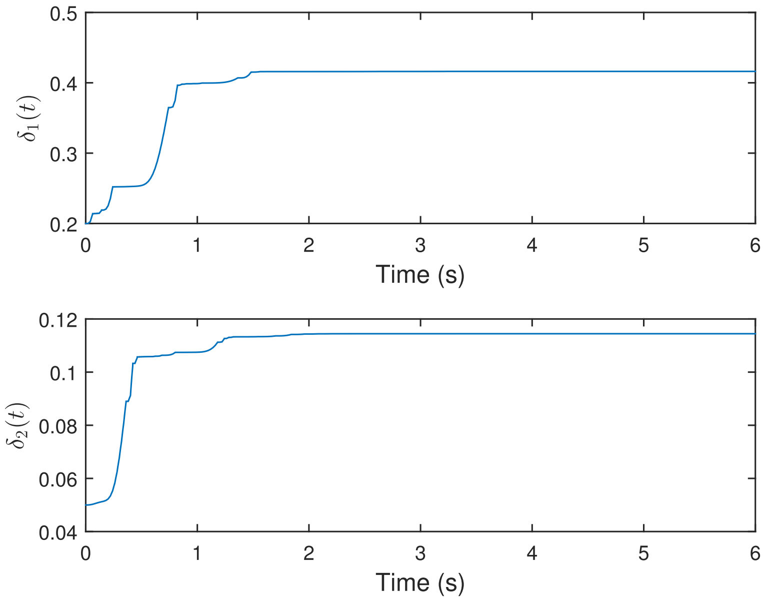

Fig. 7. The adaptive law of AETS.

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